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Technique for measuring the parameters of polarization of an ultrasonic wave

A. M. Burkhanov, K. B. Vlasov, V. V. Gudkov, and B. V. Tarasov

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Among physical phenomena consisting in variation of the polarization of a shear ulrasonic wave the acoustic analogs of the Faraday and the Cotton-Mouton effects are investigated at present (see [1] and [2] – the first theorectical papers, [3] – discovery of rotation of the polarization, and [4] –[7] – some experiments). They are observed when initially linearly polarized ultrasonic wave propagates inside a bulk specimen and are due to interaction between elastic and magnetic subsystems or conduction electrons. Quantitative characteristics of the effects are polarization parameters: ε – the ellipticity which modulus is the ratio of the minor and major ellipse axes and ϕ – the angle of rotation of the polarization plane or, more correctly, of the major ellipse axis if $\varepsilon \neq 0$. Most of recent experiments on polarization phenomena were performed with the use of phase-amplitude methods. A review of them is given in Ref. [8].

Besides, a phenomenon is considered as the acoustic analog of magneto-optic Kerr effect if variation of the polarization occurs while reflection of the wave from an interface between magnetic medium and isotropic non-magnetic one. It was predicted by Vlasov and Kuleev [9] in 1968, however, there was no papers yet about experiments in which both the parameters characterizing the polarization, ε and ϕ, were measured.

We have completed such an experiment and the results will be published soon. While performing it we found that a very small variations of a high level signal took place and came to a conclusion that amplitude variant of a technique should be more suitable here.

First amplitude technique for a precise measurement of ϕ was introduced by Boyd and Gavenda [10]. Its aplicability was limited to the case where $\varepsilon \approx 0$. Though, we developed an amplitude method free of this restriction for measuring ϕ as well as ε. A description of the technique is the subject of this paper.

The method consists of measuring the amplitude of the voltage, $V(H)$, on the receiving transducer at a certain B_1 relative to an initial $B = B_0$ using three different angles for the receiving transducer ψ with futher processing of the data with the formulas (2), (22), and (23) presented below. It can be used for investigating the acoustic analogs of the Faraday and the Cotton-Mouton effects as well.

A periodic motion of the volume element over an elliptic trajectory can be repersented with the help of amplitudes, $u^{\pm}$, and phases, $\varphi^{\pm}$, of circular elastic vibrations. Introducing a parameter

$$p = (u^-/u^+)\, e^{i(\varphi^- - \varphi^+)}, \tag{1}$$

expressions for ε and ϕ have the form:

$$\varepsilon = \frac{1-|p|}{1+|p|}, \qquad \phi = -\frac{1}{2}\,\mathrm{Im}\,[\ln(p)]\,. \tag{2}$$

Projection of the elastic vibrations to polarization direction of the receiving transducer can be written as follows:

$$\begin{aligned} u_r(t) = \mathbf{u}\cdot\mathbf{e}_r = \mathrm{Re}\Big\{ & u^+ \exp\Big[i(\omega t - \varphi^+ - \psi)\Big] \\ & + u^- \exp\Big[-i(\omega t - \varphi^- + \psi)\Big]\Big\}, \end{aligned} \tag{3}$$

where * designates the complex conjugate, $\mathbf{e}_r$ is unit vector of the direction of the polarization of the receiving transducer, ψ is the angle between this direction and the plane of incidence, ω is frequency, and t is time. u_r excite an ac voltage $V\cos(\omega t - \alpha) = \eta u_r$ (where η^2 is the coefficient of transformation of elastic vibration energy into electric field energy, and α is a phase constant). Using Eq. (3) we have

$$\begin{aligned} & \frac{V}{\eta}\,[\cos\omega t\cos\alpha + \sin\omega t\sin\alpha] \\ & \quad = \Big[u^+\cos(\varphi^+ + \psi) + u^-\cos(\varphi^- - \psi)\Big]\cos\omega t \\ & \qquad + \Big[u^+\sin(\varphi^+ + \psi) + u^-\sin(\varphi^- - \psi)\Big]\sin\omega t. \end{aligned} \tag{4}$$

Since Eq. (4) is valid for arbitrary t, it may be transformed into two equations:

$$\frac{V}{\eta}\cos\alpha = u^+\cos(\varphi^+ + \psi) + u^-\cos(\varphi^- - \psi), \tag{5}$$

$$\frac{V}{\eta}\sin\alpha = u^+\sin(\varphi^+ + \psi) + u^-\sin(\varphi^- - \psi). \tag{6}$$

Multiplying Eq. (6) by i and adding the result to Eq. (5) we obtain

$$\frac{V}{\eta}e^{i\alpha} = u^+ e^{i(\varphi^+ + \psi)} + u^- e^{i(\varphi^- - \psi)}. \tag{7}$$

The method suggested here for determining the polarization of the reflected wave consists of measuring the amplitude of the signal at a certain B_1 relative to an initial $B = B_0$ using three different angles for the receiving transducer: ψ_1, ψ_2, and ψ_3. We assume that $\varepsilon(B_0) = 0$ and $\phi(B_0) = 0$. Relevant equations for the two different values of B and three of ψ may be obtained by making the appropriate substitutions into Eq. (7). Introducing indexes $j = 0, 1$ for the two values of B and $k = 1, 2, 3$ for the three values of ψ for V_{kj}, α_{kj}, $u_j^\pm$, and $\varphi_j^\pm$ we have:

$$\frac{V_{10}}{\eta}\, e^{i\alpha_{10}} = u_0^+ e^{i\left(\varphi_0^+ + \psi_1\right)} + u_0^- e^{i\left(\varphi_0^- - \psi_1\right)}, \tag{8}$$

$$\frac{V_{11}}{\eta}\, e^{i\alpha_{11}} = u_1^+ e^{i\left(\varphi_1^+ + \psi_1\right)} + u_1^- e^{i\left(\varphi_1^- - \psi_1\right)}. \tag{9}$$

Dividing Eq. (9) by (8), we obtain

$$\frac{V_{11}}{V_{10}} e^{i(\alpha_{11} - \alpha_{10})} = F_1^+ e^{i\psi_1} + F_1^- e^{-i\psi_1}, \tag{10}$$

where

$$F_1^\pm \equiv \frac{u_1^\pm\, e^{i\varphi_1^\pm}}{u_0^+ \exp[i(\varphi_0^+ + \psi_1)] + u_0^- \exp[i(\varphi_0^- - \psi_1)]}. \tag{11}$$

Similar equations for $\psi = \psi_2$ have the form

$$\frac{V_{21}}{V_{10}} e^{i(\alpha_{21} - \alpha_{10})} \delta_2\, e^{i\lambda_2} = F_1^+\, e^{i\psi_2} + F_1^-\, e^{-i\psi_2}, \tag{12}$$

where λ_2 and δ_2 describe variations in phase and amplitude of the signal, respectively, caused by differences in transducer coupling to the sample while changing ψ from ψ_1 to ψ_2.

One more change in ψ gives the following equations in addition to (10) and (12):

$$\frac{V_{31}}{V_{10}} e^{i(\alpha_{31} - \alpha_{10})}\, \delta_3\, e^{i\lambda_3} = F_1^+\, e^{i\psi_3} + F_1^-\, e^{-i\psi_3}. \tag{13}$$

Here δ_3 and λ_3 have the same origin as δ_2 and λ_2, but correspond to changing ψ from ψ_1 to ψ_3.

After multiplying the left and right sides of Eqs. (10), (12), and (13) by their complex conjugates we obtain

$$\left(\frac{V_{11}}{V_{10}}\right)^2 = \left|F_1^+\right|^2 + \left|F_1^-\right|^2 + 2\left|F_1^+\right|\left|F_1^-\right| \cos\left(\Delta\varphi_1 + 2\psi_1\right), \tag{14}$$

$$\left(\frac{V_{21}\,\delta_2}{V_{10}}\right)^2 = \left|F_1^+\right|^2 + \left|F_1^-\right|^2 + 2\left|F_1^+\right|\left|F_1^-\right| \cos\left(\Delta\varphi_1 + 2\psi_2\right), \tag{15}$$

$$\left(\frac{V_{31}\,\delta_3}{V_{10}}\right)^2 = \left|F_1^+\right|^2 + \left|F_1^-\right|^2 + 2\left|F_1^+\right|\left|F_1^-\right| \cos\left(\Delta\varphi_1 + 2\psi_3\right), \tag{16}$$

where

$$\Delta\varphi_1 = \varphi^+(B_1) - \varphi^-(B_1), \tag{17}$$

and, due to the assumption of $\varepsilon(B_0) = 0$ and $\phi(B_0) = 0$,

$$\delta_i = \frac{V_{10}\cos\left(\psi_i\right)}{V_{i0}\cos\left(\psi_1\right)}. \tag{18}$$

These operations are necessary to remove the phase α_{kj} from our equations since amplitude is the only parameter measured in this variant of a technique. We divide both sides of Eqs. (14)–(16) by $\left|F_1^+\right|\left|F_1^-\right|$ to obtain

$$|p_1|^{-1} + |p_1| + 2\cos\left[2(\phi_1 - \psi_1)\right] = \frac{\left(V_{11}/V_{10}\right)^2}{\left|F_1^+\right|\left|F_1^-\right|}, \tag{19}$$

$$|p_1|^{-1} + |p_1| + 2\cos\left[2(\phi_1 - \psi_2)\right] = \frac{\left(V_{21}\,\delta_2/V_{10}\right)^2}{\left|F_1^+\right|\left|F_1^-\right|}, \tag{20}$$

$$|p_1|^{-1} + |p_1| + 2\cos\left[2(\phi_1 - \psi_3)\right] = \frac{\left(V_{31}\,\delta_3/V_{10}\right)^2}{\left|F_1^+\right|\left|F_1^-\right|}, \tag{21}$$

where $p_1 \equiv p(B_1)$.

Thus we have three equations with three unknowns, namely $\left|F_1^+\right|\left|F_1^-\right|$, $|p_1|$, and ϕ_1. The latter two are the parameters we are interested in and corresponding solutions of the system have the form

$$\begin{aligned}\phi_1 \; = \; & \frac{1}{2}\tan^{-1}\Big\{\Big[\left(V_{21}^2\,\delta_2^2 - V_{31}^2\,\delta_3^2\right)\cos 2\psi_1 \\ & + \left(V_{31}^2\,\delta_3^2 - V_{11}^2\right)\cos 2\psi_2 + \left(V_{11}^2 - V_{21}^2\,\delta_2^2\right)\cos 2\psi_3\Big] \\ & \times\Big[\left(V_{21}^2\,\delta_2^2 - V_{31}^2\,\delta_3^2\right)\sin 2\psi_1 + \left(V_{31}^2\,\delta_3^2 - V_{11}^2\right)\sin 2\psi_2 \\ & + \left(V_{11}^2 - V_{21}^2\,\delta_2^2\right)\sin 2\psi_3\Big]^{-1}\Big\}\end{aligned} \tag{22}$$

and

$$|p_1| \; = \; \frac{a_1}{c_1} \pm \left[\left(\frac{a_1}{c_1} \right)^2 - 1 \right]^{1/2} , \qquad (23)$$

where

$$\begin{aligned} a_1 \; &= \; V_{11}^2 \sin\left[2(\psi_2 - \psi_3)\right] + V_{21}^2 \, \delta_2^2 \sin\left[(2(\psi_3 - \psi_1)\right] \\ &\quad + V_{31}^2 \, \delta_3^2 \sin\left[2(\psi_1 - \psi_2)\right] \cos 2\phi_1, \\ c_1 \; &= \; \left(V_{21}^2 \, \delta_2^2 - V_{31}^2 \, \delta_3^2\right) \sin 2\psi_1 + \left(V_{31}^2 \, \delta_3^2 - V_{11}^2\right) \sin 2\psi_2 \\ &\quad + \left(V_{11}^2 - V_{21}^2 \, \delta_2^2\right) \sin 2\psi_3. \end{aligned}$$

The $(-)$ sign should be taken before the square root in Eq. (23), since it alone allows $|p_1| = 0$ and therefore $\varepsilon = 1$.

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